

On the contribution of twist-3 multi-gluon correlation functions to single transverse-spin asymmetry in SIDIS

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Abstract. We study the single spin asymmetry (SSA) induced by purely gluonic correlation inside a nucleon, in particular, by the three-gluon correlation functions in the transversely polarized nucleon, p^\uparrow . This contribution is embodied as a twist-3 mechanism in the collinear factorization framework and controls the SSA to be observed in the D -meson production with large transverse-momentum in semi-inclusive DIS (SIDIS), $ep^\uparrow \rightarrow eDX$. We define the relevant three-gluon correlation functions in the nucleon, and determine their complete set at the twist-3 level taking into account symmetry constraints in QCD. We derive the single-spin-dependent cross section for the D -meson production in SIDIS, taking into account all the relevant contributions at the twist-3 level. The result is obtained in a manifestly gauge-invariant form as the factorization formula in terms of the three-gluon correlation functions and reveals the five independent structures with respect to the dependence on the azimuthal angle for the produced D meson. We also demonstrate the remarkable relation between the twist-3 single-spin-dependent cross section and twist-2 cross sections for the D -meson production, as a manifestation of universal structure behind the SSA in a variety of hard processes.

1. Introduction

The single transverse-spin asymmetry (SSA) in the semi-inclusive DIS (SIDIS), $e(\ell) + p(p, S_\perp) \rightarrow e(\ell') + h(P_h) + X$, arises as T -odd effect in the cross section for the scattering of transversely polarized nucleon with momentum p and spin S_\perp , off unpolarized lepton with momentum ℓ , observing a hadron h with momentum P_h in the final state. Here, $q = \ell - \ell'$, $Q^2 = -q^2$, and $Q \gg \Lambda_{\text{QCD}}$. The SSA can be observed also in pp collisions, the pion production $p^\uparrow p \rightarrow \pi X$ [1], and the Drell-Yan and direct- γ production, $p^\uparrow p \rightarrow \gamma^{(*)} X$ [2]. Similarly as these examples, the SSA in the SIDIS requires, (i) nonzero transverse-momentum $P_{h\perp}$ originating from transverse motion of quark or gluon; (ii) nucleon helicity flip in the cut diagrams for the cross section, corresponding to the transverse polarization S_\perp ; and (iii) interaction beyond Born level to produce the interfering phase between the LHS and the RHS of the cut in those diagrams. When $P_{h\perp} \ll Q$, all (i)-(iii) may be generated nonperturbatively from the T -odd, transverse-momentum-dependent (TMD) parton distribution/fragmentation functions such as the Sivers function (see [3, 4]). By contrast, for large $P_{h\perp} \gg \Lambda_{\text{QCD}}$, (i) should come from perturbative

mechanism as the recoil from the hard (unobserved) final-state partons, while nonperturbative effects can participate in the other two, (ii) and (iii), allowing us to obtain large SSA. This is realized with the twist-3 distribution/fragmentation functions in the collinear-factorization framework. In [5], we derived the corresponding factorization formula, in the leading-order (LO) perturbative QCD, for the SSA associated with the twist-3 quark-gluon correlation inside the nucleon, and provided a practical procedure to calculate the relevant partonic hard parts manifesting their gauge invariance at the twist-3 level. A similar twist-3 mechanism for the SSA can also be generated by the purely gluonic correlations inside the nucleon. In particular, the SSA to be observed in the production of the D meson with large transverse-momentum $P_{h\perp}$ is controlled by the corresponding gluonic contributions, shown in figure 1, through the photon-gluon fusion to create a pair of c and \bar{c} quarks, one of which fragments into the D meson. In this report we discuss those gluonic effects, taking into account the nonzero masses, m_c and m_h , for the charm quark and the D meson [6]; our SSA formula for the high $P_{h\perp}$ D -meson production, $ep^\dagger \rightarrow eDX$, differs from the corresponding result in the literature [7], and the origin of the discrepancy will be clarified.

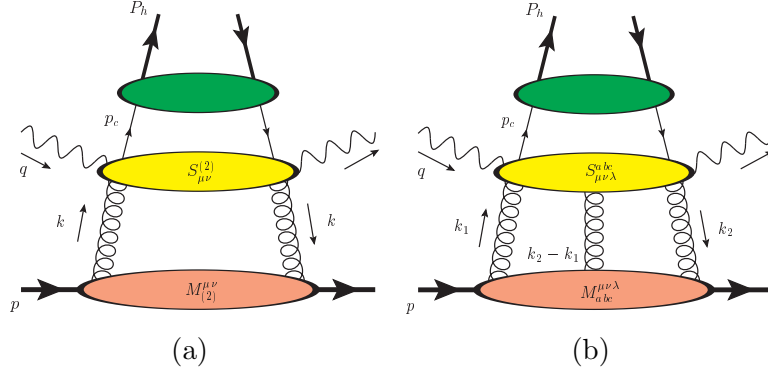


Figure 1. Generic diagrams for hadronic tensor of $ep^\dagger \rightarrow eDX$ induced by gluonic effects in the nucleon. Each one is decomposed into nucleon matrix element (lower blob), D -meson matrix element (upper blob), and partonic hard scattering part by the virtual photon through the photon-gluon fusion (middle blob).

2. Three-gluon correlation functions in the transversely polarized nucleon

Figure 1(a) gives rise to the twist-2 contribution for the unpolarized SIDIS, $ep \rightarrow eDX$, and the corresponding cross section is expressed by the factorization formula associated with the unpolarized gluon-density distribution for the nucleon,

$$G(x) = -\frac{g\beta\alpha}{x} \int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle p | F_a^{\beta n}(0) F_a^{\alpha n}(\lambda n) | p \rangle, \quad (1)$$

as the nucleon matrix element of the gauge-invariant lightcone bilocal operator using the gluon field-strength tensors, $F_a^{\alpha n}(\lambda n) \equiv F_a^{\alpha\beta}(\lambda n) n_\beta$, $F_a^{\alpha\beta} = \partial^\alpha A_a^\beta - \partial^\beta A_a^\alpha + gf_{abc} A_b^\alpha A_c^\beta$; here, $p^\mu = (p^+, 0, \mathbf{0}_\perp)$ is the nucleon momentum regarding as lightlike up to the twist-3 accuracy, $n^\mu = (0, n^-, \mathbf{0}_\perp)$ is another lightlike vector satisfying $p \cdot n = 1$, and we have suppressed the gauge-link operator which connects the field strength tensors so as to ensure the gauge invariance. Likewise, the “three-gluon distribution” functions, relevant to the SSA arising from figure 1(b), are defined through the gauge-invariant correlation function of the three field-strength tensors,

$$\begin{aligned} \mathcal{M}_{F,abc}^{\alpha\beta\gamma}(x_1, x_2) &= -g^3 \int \frac{d\lambda}{2\pi} \int \frac{d\mu}{2\pi} e^{i\lambda x_1} e^{i\mu(x_2 - x_1)} \langle p S | F_b^{\beta n}(0) F_c^{\gamma n}(\mu n) F_a^{\alpha n}(\lambda n) | p S \rangle \\ &= \frac{3}{40} d^{abc} O^{\alpha\beta\gamma}(x_1, x_2) - \frac{i}{24} f^{abc} N^{\alpha\beta\gamma}(x_1, x_2), \end{aligned} \quad (2)$$

for the nucleon with the transverse spin vector $S^\mu = (0, 0, \mathbf{S}_\perp)$, normalized as $S^2 = -\mathbf{S}_\perp^2 = -1$. This may be regarded as an extension of the quark-gluon correlation functions discussed in,

e.g., [5], with $x_{1,2} \equiv k_{1,2} \cdot n$ denoting the relevant momentum fractions of the gluons shown in figure 1(b). Here, d^{abc} and f^{abc} are the usual symmetric and anti-symmetric structure constants of the color SU(3) group, and the constraints from hermiticity, invariance under the parity and time-reversal transformations, and the permutation symmetry among the participating gluon fields allow the further decomposition in terms of the real, Lorentz-scalar functions, $O(x_1, x_2)$ and $N(x_1, x_2)$, which are both symmetric under the interchange $x_1 \leftrightarrow x_2$, such that [6]

$$O^{\alpha\beta\gamma}(x_1, x_2) = 2iM_N \left[O(x_1, x_2) g^{\alpha\beta} \epsilon^{\gamma p n S} + O(-x_2, x_1 - x_2) g^{\beta\gamma} \epsilon^{\alpha p n S} + O(x_2 - x_1, -x_1) g^{\gamma\alpha} \epsilon^{\beta p n S} \right], \quad (3)$$

and similarly for $N^{\alpha\beta\gamma}(x_1, x_2)$, at the twist-3 accuracy; here, $\epsilon^{\gamma p n S} \equiv \epsilon^{\gamma\lambda\mu\nu} p_\lambda n_\mu S_\nu$. $O(x_1, x_2)$ and $N(x_1, x_2)$ are defined as dimensionless, accompanying the nucleon mass M_N in (3), and are associated, respectively, with the C -odd and C -even combinations of the three gluon operators in (2), satisfying $O(x_1, x_2) = O(-x_1, -x_2)$ and $N(x_1, x_2) = -N(-x_1, -x_2)$. Indeed, $O(x_1, x_2)$ and $N(x_1, x_2)$ constitute a complete set for expressing gluonic correlation effects inside the nucleon at the twist-3 level. We note that the other two functions, in addition to $O(x_1, x_2)$ and $N(x_1, x_2)$, were introduced in [8] to constitute a basis of three-gluon functions at the twist-3 level, but this basis proves to be redundant, i.e., the two functions $O(x_1, x_2)$ and $N(x_1, x_2)$ form a genuine complete set [6]. The authors of [9] also pointed out that there are only two independent pure gluonic functions at twist-3, in the study of the evolution equations for the twist-3 distributions.

3. Collinear expansion and gauge invariance for the three-gluon correlation effects

We calculate the hadronic tensor $W^{\mu\nu}(p, q, P_h)$, represented by figure 1. We are interested in the contribution in which the c and \bar{c} quarks are created through the photon-gluon fusion process and one of them fragments into a D (\bar{D}) meson, so that the corresponding fragmentation function $D(z)$, with z being the relevant momentum fraction, is factorized from $W_{\mu\nu}$ as

$$W_{\mu\nu}(p, q, P_h) = \int \frac{dz}{z^2} D(z) w_{\mu\nu}(p, q, p_c), \quad (4)$$

where the summation over the c and \bar{c} quark contributions is implicit, and p_c is the momentum of the c (or \bar{c}) quark with mass m_c ; using the lightlike vector w of $O(1/Q)$ satisfying $P_h \cdot w = 1$, we have, $p_c^\mu = P_h^\mu/z + r w^\mu$, with $r = (m_c^2 z - m_h^2/z)/2$ to satisfy $p_c^2 = m_c^2$, and, at the leading twist-2 accuracy for the quark-fragmentation process, we set $w^\mu = p^\mu/(P_h \cdot p)$. We also note that, in the LO with respect to the QCD coupling constant for the partonic hard scattering parts, the contribution of figure 1(a) can not give rise to the SSA, due to the symmetry properties of the correlation functions of two gluon fields in the polarized nucleon (see Appendix A in [6]). So we shall focus on the analysis of figure 1(b). We work in Feynman gauge and apply the collinear expansion to the hard scattering part in figure 1(b), keeping all the terms contributing up to the relevant accuracy of twist-3. We perform the corresponding calculation on the basis of generalization of our previous formulation [5] that was developed for the case of the SSA for the pion production induced by the quark-gluon correlation inside the nucleon. For simplicity, we shall henceforth write $w_{\mu\nu}(p, q, p_c)$ of (4) as $w(p, q, p_c)$, omitting the indices for virtual photon.

The contribution from figure 1(b) to $w(p, q, p_c)$ can be written as

$$w(p, q, p_c) = \int \frac{d^4 k_1}{(2\pi)^4} \int \frac{d^4 k_2}{(2\pi)^4} S_{\mu\nu\lambda}^{abc}(k_1, k_2, q, p_c) M_{abc}^{\mu\nu\lambda}(k_1, k_2), \quad (5)$$

where $S_{\mu\nu\lambda}^{abc}(k_1, k_2, q, p_c)$ is the partonic hard scattering part (middle blob of figure 1(b)), and

$$M_{abc}^{\mu\nu\lambda}(k_1, k_2) = g \int d^4 \xi \int d^4 \eta e^{ik_1 \xi} e^{i(k_2 - k_1) \eta} \langle pS | A_b^\nu(0) A_c^\lambda(\eta) A_a^\mu(\xi) | pS \rangle \quad (6)$$

is the corresponding nucleon matrix element (lower blob), including one QCD coupling constant g . In (5), a real contribution relevant to the spin-dependent cross section occurs from an imaginary part of the color-projected hard part $C_{\pm}^{bca} S_{\mu\nu\lambda}^{abc}(k_1, k_2, q, p_c)$ with $C_+^{bca} = if^{bca}$ and $C_-^{bca} = d^{bca}$, since $C_{\pm}^{bca} M_{abc}^{\mu\nu\lambda}(k_1, k_2)$ are pure imaginary quantities [6]. This means that only the pole contribution produced by an internal propagator in the hard part can give rise to SSA.

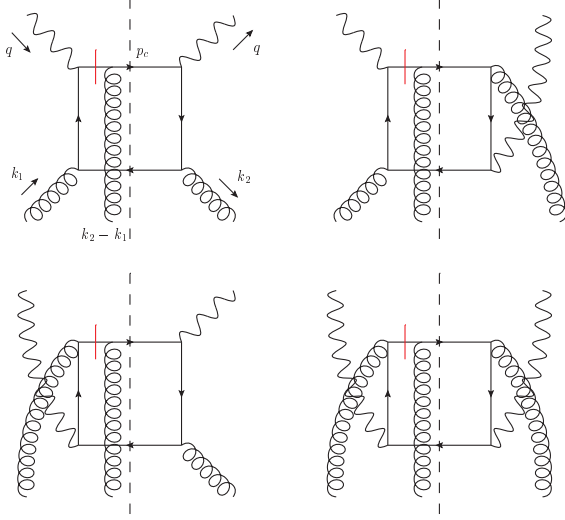


Figure 2. Feynman diagrams for the partonic hard part in figure 1(b), representing the photon-gluon fusion subprocesses that give rise to the “surviving” pole contribution for $ep^\uparrow \rightarrow eDX$ in the LO with respect to the QCD coupling constant. The short bar on the internal c -quark line indicates that the pole part is to be taken from that propagator. In the text, momenta are assigned as shown in the upper-left diagram, where p_c denotes the momentum of the c -quark fragmenting into the D -meson in the final state. The mirror diagrams also contribute.

In the LO with respect to the QCD coupling constant, we find that the diagrams shown in figure 2, together with their mirror diagrams, give rise to the “surviving” pole contributions; here, the quark propagator with a short bar produces the corresponding pole contribution. The other pole contributions turn out to cancel among themselves after summing the contributions of all the leading-order diagrams for (5). With the assignment of the momenta k_1 and k_2 of gluons as shown in figure 2, the condition for those poles is given by $(p_c - k_2 + k_1)^2 - m_c^2 = 0$. After we perform the collinear expansion and reach the collinear limit, $k_i \rightarrow x_i p$ ($i = 1, 2$), with x_1, x_2 and $x_2 - x_1$ representing the longitudinal momentum fractions of the relevant three gluons, this condition reduces to $x_1 = x_2$ and hence a pole of such type is referred to as the soft-gluon pole (SGP). In the following, we assume that $S_{\mu\nu\lambda}^{abc}(k_1, k_2, q, p_c)$ in (5) represents the sum of the contributions of the diagrams in figure 2 and their mirror diagrams, in which the barred propagator is replaced by its pole contribution. We write the hard part $S_{\mu\nu\lambda}^{abc}(k_1, k_2, q, p_c)$ as $S_{\mu\nu\lambda}(k_1, k_2)$, for simplicity, suppressing the color indices a, b, c and the momenta q and p_c , and, correspondingly, we write $M_{abc}^{\mu\nu\lambda}(k_1, k_2)$ as $M^{\mu\nu\lambda}(k_1, k_2)$.

To perform the collinear expansion, as usual, in the “hadron frame” [5] with $q^\mu = (q^0, \vec{q}) = (0, 0, 0, -Q)$, where both the virtual photon and the initial nucleon move along the z -axis, we decompose the relevant gluon momenta k_i ($i = 1, 2$) as $k_i^\mu = (k_i \cdot n)p^\mu + (k_i \cdot p)n^\mu + k_\perp^\mu \equiv x_i p^\mu + \omega_\nu^\mu k_\nu^\mu$, with $\omega_\nu^\mu \equiv g_\nu^\mu - p^\mu n_\nu$. Since $p^\mu \sim g_+^\mu Q$ in the hadron frame and thus the component along p^μ gives the leading contribution with respect to the hard scale Q , we expand $S_{\mu\nu\lambda}(k_1, k_2)$ around $k_i = x_i p$. Expressing also the gluon field A^α as $A^\alpha = (p^\alpha n_\kappa + \omega_\kappa^\alpha) A^\kappa = p^\alpha n \cdot A + \omega_\kappa^\alpha A^\kappa$, we note that, in the matrix element $M^{\mu\nu\lambda}(k_1, k_2)$ of (6), the components associated with the second term $\omega_\kappa^\alpha A^\kappa$ give rise to the contributions suppressed by $\sim 1/Q$ or more, compared with the corresponding contribution due to the first term, $p^\alpha n \cdot A$ (see, e.g., [5]). We organize the integrand of (5) according to the order counting based on those decompositions, keeping the terms necessary in the twist-3 accuracy. For this purpose, we need the Taylor expansion of the hard partonic subprocesses up to the terms with the third-order derivative, as $S_{\mu\nu\lambda}(k_1, k_2) \simeq S_{\mu\nu\lambda}(x_1, x_2) + \omega_\kappa^\alpha k_1^\kappa \partial S_{\mu\nu\lambda}(k_1, k_2) / \partial k_1^\alpha|_{k_i=x_i p} + \dots +$

$(1/6)\omega_\kappa^\alpha k_2^\kappa \omega_\tau^\beta k_2^\tau \omega_\sigma^\gamma k_2^\sigma \partial^3 S_{\mu\nu\lambda}(k_1, k_2)/\partial k_2^\alpha \partial k_2^\beta \partial k_2^\gamma \Big|_{k_i=x_i p}$, where $S_{\mu\nu\lambda}(x_1, x_2) \equiv S_{\mu\nu\lambda}(x_1 p, x_2 p)$, because the terms with the third derivatives could produce the contributions that behave as the same order as the term, $S_{\alpha\beta\gamma}(x_1, x_2)\omega_\kappa^\alpha \omega_\tau^\beta \omega_\sigma^\gamma M^{\kappa\tau\sigma}(k_1, k_2)$, arising in the collinear expansion of the integrand of (5): If we substitute $S_{\alpha\beta\gamma}(x_1, x_2)\omega_\kappa^\alpha \omega_\tau^\beta \omega_\sigma^\gamma M^{\kappa\tau\sigma}(k_1, k_2)$ directly into the integrand of (5) and perform the integrals over k_1 and k_2 , the result produces the contribution, which is associated with the hard scattering between the three physical gluons from the nucleon and behaves as the same order as the formal convolution of $S_{\alpha\beta\gamma}(x_1, x_2)$ with the twist-3 correlation functions in (2); this corresponds to a quantity of twist-3. Thus, our collinear expansion produces lots of terms, each of which is not gauge invariant, and it would look hopeless to reorganize those into a form of the convolution with only the gauge-invariant correlation functions $O^{\alpha\beta\gamma}(x_1, x_2)$ and $N^{\alpha\beta\gamma}(x_1, x_2)$ of (2) used.

However, as was the case for the pion production associated with the twist-3 quark-gluon correlation functions [5], great simplification occurs due to Ward identities satisfied by the corresponding partonic hard-scattering function, $S_{\mu\nu\lambda}(k_1, k_2)$. To derive the Ward identities, we note that the contribution to $S_{\mu\nu\lambda}(k_1, k_2)$ from each diagram in figure 2 all contains the two delta functions, $\delta((k_2 + q - p_c)^2 - m_c^2)$ and $\delta((p_c + k_1 - k_2)^2 - m_c^2)$, representing the on-shell conditions associated, respectively, with the final-state cut on the unobserved \bar{c} -quark line and with the unpinched pole contribution of the barred propagator. Also, in each diagram in figure 2, the c -quark line fragmenting into the final-state D -meson is on-shell (see the discussion below (4)). Due to these on-shell conditions for the diagrams in figure 2 and the similar conditions for their mirror diagrams, $S_{\mu\nu\lambda}(k_1, k_2)$ satisfies the Ward identities [6],

$$k_1^\mu S_{\mu\nu\lambda}(k_1, k_2) = 0, \quad k_2^\nu S_{\mu\nu\lambda}(k_1, k_2) = 0, \quad (k_2 - k_1)^\lambda S_{\mu\nu\lambda}(k_1, k_2) = 0, \quad (7)$$

which can be used to reorganize our collinear expansion. Indeed, combined with another relation,

$$\left. \frac{\partial S_{\mu\nu\lambda}(k_1, k_2) p^\lambda}{\partial k_{1\perp}^\sigma} \right|_{k_i=x_i p} = - \left. \frac{\partial S_{\mu\nu\lambda}(k_1, k_2) p^\lambda}{\partial k_{2\perp}^\sigma} \right|_{k_i=x_i p}, \quad (8)$$

which can be derived by direct inspection of the diagrams in figure 2 and their mirror diagrams, we find [6] that all the gauge-noninvariant terms in our collinear expansion vanish or cancel among themselves, and the remaining terms can be expressed in a gauge-invariant form, as the factorization formula in terms of the three-gluon correlation functions of twist-3, defined in (2),

$$w(p, q, p_c) = \int \frac{dx_1}{x_1} \int \frac{dx_2}{x_2} \left. \frac{\partial S_{\mu\nu\lambda}^{abc}(k_1, k_2, q, p_c) p^\lambda}{\partial k_2^\sigma} \right|_{k_i=x_i p} \omega_\alpha^\mu \omega_\beta^\nu \omega_\gamma^\sigma \mathcal{M}_{F,abc}^{\alpha\beta\gamma}(x_1, x_2), \quad (9)$$

up to the higher-order corrections beyond the present accuracy. Here, we have restored the color indices a, b, c as well as momentum variables q, p_c , which were associated with the hard-scattering function in (5). Thus, the *total twist-3* contribution to $w(p, q, p_c)$, relevant to SSA, proves to be expressed solely in terms of the three-gluon functions $O(x_1, x_2)$ and $N(x_1, x_2)$ (see (3)).

When calculating the hard part in (9), $\partial S_{\mu\nu\lambda}^{abc}(k_1, k_2, q, p_c) p^\lambda / \partial k_2^\sigma \Big|_{k_i=x_i p}$, the derivative with respect to k_2^σ can hit the delta functions, $\delta((k_2 + q - p_c)^2 - m_c^2)$ and $\delta((p_c + k_1 - k_2)^2 - m_c^2)$, which are involved in $S_{\mu\nu\lambda}^{abc}(k_1, k_2, q, p_c) p^\lambda$ as mentioned above (7). Such derivatives of these delta functions can be reexpressed by the derivatives with respect to x_2 , and then be treated by integration by parts, giving rise to the derivative of the three-gluon functions $O(x_1, x_2)$ and $N(x_1, x_2)$. After such manipulations, we have $\delta((p_c + k_1 - k_2)^2 - m_c^2) \Big|_{k_i=x_i p} = (1/2p_c \cdot p) \delta(x_1 - x_2)$, manifesting the SGP contribution, while another delta function becomes $(\hat{x} \equiv x_{bj}/x, \hat{z} \equiv z_f/z, q_T \equiv P_{h\perp}/z_f \text{ with } x_{bj} = Q^2/(2p \cdot q) \text{ and } z_f = p \cdot P_h/p \cdot q, \text{ as usual})$

$$\delta((k_2 + q - p_c)^2 - m_c^2) \Big|_{k_2=xp} = \frac{1}{\hat{z}Q^2} \delta\left(\frac{q_T^2}{Q^2} - \left(\frac{1}{\hat{x}} - 1\right)\left(\frac{1}{\hat{z}} - 1\right) + \frac{m_c^2}{\hat{z}^2 Q^2}\right), \quad (10)$$

implying $x > x_{bj}$; the similar contributions arise also from the corresponding mirror diagrams. Therefore, evaluating the SGP contributions in the hard part in (9), the resulting SSA proves to receive contributions associated with $O(x, x)$, $O(x, 0)$, $N(x, x)$, $N(x, 0)$, and also their derivatives with respect to x . In particular, the partonic hard part associated with $O(x, x)$ ($dO(x, x)/dx$) is different from the hard part associated with $O(x, 0)$ ($dO(x, 0)/dx$), due to the difference in the tensor structures among the three terms in the RHS of (3). Similarly, the partonic hard part for $N(x, x)$ ($dN(x, x)/dx$) is different from that for $N(x, 0)$ ($dN(x, 0)/dx$).

Here, we make a brief comment on the calculation presented in [7] for the same phenomenon, i.e., for the twist-3 mechanism to the SSA in SIDIS, $ep^\dagger \rightarrow eDX$. The calculation by Kang and Qiu in [7] started from a factorization formula that was proposed as a straightforward extension of the corresponding factorization formula for the SSA in the pion production, associated with the twist-3 quark-gluon correlation functions, but Kang-Qiu's factorization formula did not manifest gauge invariance, nor permutation symmetry among the participating gluons (see equations (26) and (27) in [7]). Assuming gauge invariance in their factorization formula, Kang and Qiu claimed that $w(p, q, p_c)$ can be eventually calculated with the following formula,

$$\int \frac{dx_1}{x_1} \int \frac{dx_2}{x_2} \frac{\partial S_{\mu\nu\lambda}^{abc}(k_1, k_2, q, p_c) p^\lambda g_\perp^{\mu\nu}}{\partial k_{2\perp}^\sigma} \Big|_{k_i=x_i p} \omega_\gamma^\sigma g_{\alpha\beta} \mathcal{M}_{F,abc}^{\alpha\beta\gamma}(x_1, x_2), \quad (11)$$

in the notation used in this report, where $g_\perp^{\mu\nu} = g^{\mu\nu} - p^\mu n^\nu - p^\nu n^\mu = -S^\mu S^\nu - \epsilon^{\mu p n S} \epsilon^{\nu p n S}$, and we can make the replacement $\omega_\gamma^\sigma \rightarrow g_{\perp\gamma}^\sigma$, up to the irrelevant corrections of twist-4 and higher. Combined with the property, $S_\gamma g_{\alpha\beta} \mathcal{M}_{F,abc}^{\alpha\beta\gamma}(x_1, x_2) = 0$, which follows from (2), (3), we see that the three-gluon correlation functions involved in (11) are expressed by the two types of functions of x , $T_G^{(\pm)}(x, x)$, and their derivative, after evaluating the SGP at $x_1 = x_2 \equiv x$, where

$$T_G^{(\pm)}(x, x) = \int \frac{dy_1^- dy_2^-}{2\pi} e^{ixp^+ y_1^-} \frac{1}{xp^+} g_{\beta\alpha} \epsilon_{S\gamma np} \langle pS | C_\pm^{bca} F_b^{\beta+}(0) F_c^{\gamma+}(y_2^-) F_a^{\alpha+}(y_1^-) | pS \rangle, \quad (12)$$

with $C_+^{bca} = if^{bca}$, $C_-^{bca} = d^{bca}$; in contrast to our result mentioned above, the contributions $T_G^{(\pm)}(x, 0)$, $dT_G^{(\pm)}(x, 0)/dx$, associated with unequal arguments, do not arise. The functions (12) are given by the contraction of (2) with the particular tensor $g_{\beta\alpha} \epsilon_{S\gamma np}$. Using (3), we have

$$\frac{xg}{2\pi} T_G^{(+)}(x, x) = -4M_N [N(x, x) - N(x, 0)], \quad \frac{xg}{2\pi} T_G^{(-)}(x, x) = -4M_N [O(x, x) + O(x, 0)]. \quad (13)$$

Thus, the twist-3 SSA obtained in [7] implies the same partonic hard parts for $O(x, x)$ and $O(x, 0)$ (for $dO(x, x)/dx$ and $dO(x, 0)/dx$), and similarly for $N(x, x)$ and $-N(x, 0)$ (for $dN(x, x)/dx$ and $-dN(x, 0)/dx$); clearly, such result contradicts with the above-mentioned result based on our complete formula (9). It is straightforward to see that, if the tensor structure of the three-gluon correlation function $\mathcal{M}_{F,abc}^{\alpha\beta\gamma}(x_1, x_2)$ of (2) were assumed to be given by only one structure, $g^{\alpha\beta} \epsilon^{\gamma p n S}$, our formula (9) would reduce to the formula (11), up to the corrections of twist-4 and higher. However, such assumption contradicts with the permutation symmetry required by the Bose statistics of the gluon, as discussed in section 2 and represented in (3).

4. Result for the twist-3 single-spin-dependent cross section for $ep^\dagger \rightarrow eDX$

We calculate the hadronic tensor (4) using our factorization formula (9), and evaluate the contraction $L^{\mu\nu} W_{\mu\nu}$ with the leptonic tensor for the unpolarized electron, $L_{\mu\nu} = 2(\ell_\mu \ell'_\nu + \ell_\nu \ell'_\mu) - Q^2 g_{\mu\nu}$. The result is decomposed according to the dependence on the azimuthal angles ϕ and Φ_S for the initial-lepton's 3-momentum $\vec{\ell}$ and the nucleon's spin vector \vec{S} , respectively, measured

from the axis along the transverse-momentum $\vec{P}_{h\perp}$ in the hadron frame, and, through standard manipulations, leads to the spin-dependent, differential cross section for $ep^\uparrow \rightarrow eDX$ [6]:

$$\begin{aligned} \frac{d^5 \Delta \sigma}{[d\omega]} &= \frac{\alpha_{em}^2 \alpha_s e_c^2 M_N}{8\pi x_{bj}^2 S_{ep}^2 Q^2} \left(\frac{-\pi}{2} \right) \sum_k \mathcal{A}_k \mathcal{S}_k \int \frac{dx}{x} \int \frac{dz}{z} \delta \left(\frac{q_T^2}{Q^2} - \left(1 - \frac{1}{\hat{x}} \right) \left(1 - \frac{1}{\hat{z}} \right) + \frac{m_c^2}{\hat{z}^2 Q^2} \right) \sum_{a=c,\bar{c}} D_a(z) \\ &\times \left(\delta_a \left\{ \left[\frac{dO(x,x)}{dx} - \frac{2O(x,x)}{x} \right] \Delta \hat{\sigma}_k^1 + \left[\frac{dO(x,0)}{dx} - \frac{2O(x,0)}{x} \right] \Delta \hat{\sigma}_k^2 + \frac{O(x,x)}{x} \Delta \hat{\sigma}_k^3 + \frac{O(x,0)}{x} \Delta \hat{\sigma}_k^4 \right\} \right. \\ &\left. + \left[\frac{dN(x,x)}{dx} - \frac{2N(x,x)}{x} \right] \Delta \hat{\sigma}_k^1 - \left[\frac{dN(x,0)}{dx} - \frac{2N(x,0)}{x} \right] \Delta \hat{\sigma}_k^2 + \frac{N(x,x)}{x} \Delta \hat{\sigma}_k^3 - \frac{N(x,0)}{x} \Delta \hat{\sigma}_k^4 \right), \end{aligned} \quad (14)$$

where $S_{ep} = (p+\ell)^2$, we use the shorthand notation, $[d\omega] \equiv dx_{bj} dQ^2 dz_f dq_T^2 d\phi$, for the differential elements, and the summation \sum_k implies that the subscript k runs over 1, 2, 3, 4, 8, 9, with

$$\begin{aligned} \mathcal{A}_1 &= 1 + \cosh^2 \psi, & \mathcal{A}_2 &= -2, & \mathcal{A}_3 &= -\cos \phi \sinh 2\psi, & \mathcal{A}_4 &= \cos 2\phi \sinh^2 \psi, \\ \mathcal{A}_8 &= -\sin \phi \sinh 2\psi, & \mathcal{A}_9 &= \sin 2\phi \sinh^2 \psi, & (\cosh \psi &\equiv \frac{2x_{bj} S_{ep}}{Q^2} - 1), \end{aligned} \quad (15)$$

and \mathcal{S}_k defined as $\mathcal{S}_k = \sin \Phi_S$ for $k = 1, 2, 3, 4$ and $\mathcal{S}_k = \cos \Phi_S$ for $k = 8, 9$. The quark-flavor index a can, in principle, be c and \bar{c} , with $\delta_c = 1$ and $\delta_{\bar{c}} = -1$, so that the cross section for the \bar{D} -meson production $ep^\uparrow \rightarrow e\bar{D}X$ can be obtained by a simple replacement of the fragmentation function to that for the \bar{D} meson, $D_a(z) \rightarrow \bar{D}_a(z)$. The delta function of (10) appears, α_{em} is the fine-structure constant, $\alpha_s = g^2/(4\pi)$ is the strong coupling constant, and $e_c = 2/3$ is the electric charge of the c -quark. Partonic hard parts $\Delta \hat{\sigma}_k^i$ depend on m_c as well as other partonic variables; for the explicit formulae of $\Delta \hat{\sigma}_k^i$, we refer the readers to [6]. We find that $\Delta \hat{\sigma}_k^1 \neq \Delta \hat{\sigma}_k^2$, $\Delta \hat{\sigma}_k^3 \neq \Delta \hat{\sigma}_k^4$, while the hard parts arising in the second and third lines in (14) are common.

The single-spin-dependent cross section (14) can be decomposed into the five structure functions, based on the different dependences on the azimuthal angles Φ_S and ϕ through the above-mentioned explicit forms of \mathcal{A}_k and \mathcal{S}_k . The five independent azimuthal structures of this type have been observed also in the twist-3 single-spin-dependent cross section for $ep^\uparrow \rightarrow e\pi X$, generated from the quark-gluon correlation functions, as presented in [10, 11]. Introducing the azimuthal angles ϕ_h and ϕ_S of the hadron plane and the nucleon's spin vector \vec{S} , respectively, as measured from the *lepton plane*, they are connected to the above Φ_S and ϕ as $\Phi_S = \phi_h - \phi_S$, $\phi = \phi_h$, and one may express (14) as the superposition of five sine modulations,

$$\frac{d^5 \Delta \sigma}{[d\omega]} = f_1 \sin(\phi_h - \phi_S) + f_2 \sin(2\phi_h - \phi_S) + f_3 \sin \phi_S + f_4 \sin(3\phi_h - \phi_S) + f_5 \sin(\phi_h + \phi_S), \quad (16)$$

with the corresponding structure functions f_1, f_2, \dots, f_5 . A similar form of five independent azimuthal dependences in the SSA was obtained in the TMD approach [3] for small- $P_{h\perp}$ regions.

It is worth comparing (14) with the unpolarized cross section for SIDIS, $ep \rightarrow eDX$, which can be obtained straightforwardly by applying the above-developed formalism to extract the twist-2 contribution from figure 1(a), generated from the unpolarized gluon-density distribution (1), as

$$\frac{d^5 \sigma^{\text{unpol}}}{[d\omega]} = \frac{\alpha_{em}^2 \alpha_s e_c^2}{32\pi x_{bj}^2 S_{ep}^2 Q^2} \sum_k \mathcal{A}_k \int \frac{dx}{x} \int \frac{dz}{z} \delta \left(\frac{q_T^2}{Q^2} - \left(1 - \frac{1}{\hat{x}} \right) \left(1 - \frac{1}{\hat{z}} \right) + \frac{m_c^2}{\hat{z}^2 Q^2} \right) \sum_{a=c,\bar{c}} D_a(z) G(x) \hat{\sigma}_k^U, \quad (17)$$

in the same notation as in (14), with $\hat{\sigma}_8^U = \hat{\sigma}_9^U = 0$. Comparing the explicit form [6] of the LO partonic hard cross sections $\hat{\sigma}_k^U$ with the partonic hard parts $\Delta \hat{\sigma}_k^1$ in (14), associated with the derivatives, $dO(x,x)/dx$ and $dN(x,x)/dx$, of the three-gluon functions, one finds the following relations between the partonic hard parts at the twist-3 level and those at the twist-2 level:

$$\Delta \hat{\sigma}_k^1 = \frac{2q_T \hat{x}}{Q^2(1 - \hat{z})} \hat{\sigma}_k^U. \quad (18)$$

The other partonic cross sections at the twist-3 level, $\Delta\hat{\sigma}_k^i$ ($i = 2, 3, 4$) in (14), are also related to the partonic hard scattering parts at the twist-2 level, although, unlike (18), the corresponding relations cannot be manifested by direct comparison between their explicit formulae. We find that all these remarkable relations are consequences of the “master formula”,

$$w(p, q, p_c) = -i\pi \int \frac{dx}{x^2} \frac{\partial \mathcal{H}_{\mu\nu}^{abc}(xp, q, p_c)}{\partial p_{c\perp}^\sigma} \omega_\alpha^\mu \omega_\beta^\nu \omega_\gamma^\sigma \mathcal{M}_{F,abc}^{\alpha\beta\gamma}(x, x), \quad (19)$$

which allows us to calculate (9) using the partonic hard part $\mathcal{H}_{\mu\nu}^{abc}(xp, q, p_c)$ for the $2 \rightarrow 2$ Born subprocess, which coincides with the LO contribution to the hard part in figure 1(a), up to the color structure. The proof of (19) will be presented elsewhere. The similar master formula was derived [10, 12] for various processes associated with twist-3 quark-gluon correlation functions.

5. Summary

We have investigated the SSA for the D -meson production in SIDIS, generated from the twist-3 three-gluon correlation functions for the nucleon. We first showed that there are only two independent three-gluon correlation functions of twist-3, $O(x_1, x_2)$ and $N(x_1, x_2)$, which correspond to two possible ways to construct color-singlet combination composed of three active gluons. Then, we have formulated the method for calculating the twist-3 single-spin-dependent cross section generated from the three-gluon correlations. Our formulation is based on a systematic analysis of the relevant diagrams in the Feynman gauge and gives all the contribution to the cross section at the twist-3 level in the LO in perturbative QCD, guaranteeing the gauge invariance of the result. As a result, the SSA occurs as the pole contribution of an internal propagator in the partonic hard-scattering subprocess, and the corresponding SGP contribution leads to the SSA expressed in terms of the four types of gluonic functions of the relevant momentum fraction x : $O(x, x)$, $O(x, 0)$, $N(x, x)$ and $N(x, 0)$. We find that all these four-types of functions and their derivatives with respect to x contribute to the final form of the single-spin-dependent cross section, generating five independent structures on relevant azimuthal angles. We also mentioned the master formula that manifests universal structure behind the SGP contributions, allowing us to relate the $3 \rightarrow 2$ subprocess relevant for the twist-3 level to the $2 \rightarrow 2$ subprocess for the twist-2 level. Our formalism developed for the twist-3 mechanism of the SSA arising from multi-gluon correlations, and also the above features revealed in $ep^\uparrow \rightarrow eDX$, apply to the SSA in the other processes, like A_N in $p^\uparrow p \rightarrow hX$ ($h = \pi, K, D$, etc.) [13].

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